Automata-Based Verification

We consider a system consisting of three concurrent processes P, Q_1, and Q_2:

- Process P is a lock, which can be modeled by the following Labelled Transition System, whose initial state is s_0:

$$P: \qquad s_0 \xrightarrow{\text{Lock}} s_1 \xrightarrow{\text{Unlock}} s_0$$

- Processes Q_1 and Q_2 are similar processes, each attemping to execute a sequence of actions "$\text{Open}_1; \text{Close}_1$" or "$\text{Open}_2; \text{Close}_2$", respectively. They use the lock in order to guarantee that they execute their sequence of actions atomically, i.e., without interleaving with that of the other process. Q_1 and Q_2 can be modeled as Labelled Transition Systems as follows, where the initial states are respectively $t_{1,0}$ and $t_{2,0}$:

$$Q_i \ (i \in \{1,2\}): \qquad t_{i,0} \xrightarrow{\text{Lock}} t_{i,1} \xrightarrow{\text{Open}_i} t_{i,2} \xrightarrow{\text{Close}_i} t_{i,3} \xrightarrow{\text{Unlock}} t_{i,0}$$

- The whole system is defined as the parrallel composition $P \otimes_{\{\text{Lock,Unlock}\}} (Q_1 \otimes_{\emptyset} Q_2)$.

1. Figure 1 (page 2) contains all states of the product $Q_1 \otimes_{\emptyset} Q_2$. Complete this figure by circling the initial state, and by adding the transitions of $Q_1 \otimes_{\emptyset} Q_2$.
2. Draw the (reachable part of the) product $P \otimes_{\{\text{Lock,Unlock}\}} (Q_1 \otimes_{\emptyset} Q_2)$.

$t_{1,0}, t_{2,0}$	$t_{1,1}, t_{2,0}$	$t_{1,2}, t_{2,0}$	$t_{1,3}, t_{2,0}$
$t_{1,0}, t_{2,1}$	$t_{1,1}, t_{2,1}$	$t_{1,2}, t_{2,1}$	$t_{1,3}, t_{2,1}$
$t_{1,0}, t_{2,2}$	$t_{1,1}, t_{2,2}$	$t_{1,2}, t_{2,2}$	$t_{1,3}, t_{2,2}$
$t_{1,0}, t_{2,3}$	$t_{1,1}, t_{2,3}$	$t_{1,2}, t_{2,3}$	$t_{1,3}, t_{2,3}$

Figure 1: Product $Q_1 \otimes_\emptyset Q_2$ to be completed