# Programming Language Semantics and Compiler Design

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# Midterm Exam of Monday, October 20, 2025

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- **Duration**: 1h15.
- Authorized documents up to 3 double-sided sheets of A4 paper.
- Any electronic device is forbidden.
- The grading scale is indicative.
- Exercises are **independent**.
- **The care of your submission will be taken into account.**
- Read each exercise till the end before answering.
- **Indicate your group number on your submission.**
- If you don't know how to answer a question, you may assume the result and proceed with the next question.
- The maximal grade is 20/20.

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## Answer of exercise 1

We use the following abbreviations:

$$\begin{array}{rcl} b & = & x + y > 0 \\ S_0 & = & S_1;\ S_2 \\ S_1 & = & x := x + 1 \\ S_2 & = & y := y - 2 \end{array}$$

1.

$$\dfrac{\dfrac{\dfrac{}{(S_1, [x \mapsto 1, y \mapsto 0]) \to [x \mapsto 2, y \mapsto 0]} \quad \dfrac{}{(S_2, [x \mapsto 2, y \mapsto 0]) \to [x \mapsto 2, y \mapsto -2]}}{(S_0, [x \mapsto 1, y \mapsto 0]) \to [x \mapsto 2, y \mapsto -2]} \quad \dfrac{}{(\texttt{while}\ b\ \texttt{do}\ S_0\ \texttt{od}, [x \mapsto 2, y \mapsto -2]) \to [x \mapsto 2, y \mapsto -2]}}{(\texttt{while}\ b\ \texttt{do}\ S_0\ \texttt{od}, [x \mapsto 1, y \mapsto 0]) \to [x \mapsto 2, y \mapsto -2]}$$

2.

$$\begin{array}{rcl} (\texttt{while}\ b\ \texttt{do}\ S_0\ \texttt{od}, [x \mapsto 1, y \mapsto 0]) & \Rightarrow & (S_0;\ \texttt{while}\ b\ \texttt{do}\ S_0\ \texttt{od}, [x \mapsto 1, y \mapsto 0]) \\ & \Rightarrow & (S_2;\ \texttt{while}\ b\ \texttt{do}\ S_0\ \texttt{od}, [x \mapsto 2, y \mapsto 0]) \\ & \Rightarrow & (\texttt{while}\ b\ \texttt{do}\ S_0\ \texttt{od}, [x \mapsto 2, y \mapsto -2])) \\ & \Rightarrow & [x \mapsto 2, y \mapsto -2] \end{array}$$

## Answer of exercise 2

1.

$$\dfrac{(S_0, \sigma) \to \sigma'}{(\texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od}, \sigma) \to \sigma'} \text{ if } \mathcal{B}[b](\sigma') = \mathbf{ff}$$

$$\dfrac{(S_0, \sigma) \to \sigma' \quad (\texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od}, \sigma') \to \sigma''}{(\texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od}, \sigma) \to \sigma''} \text{ if } \mathcal{B}[b](\sigma') = \mathbf{tt}$$

2. The "..." symbol must be replaced by the statement "$S_0;\ \texttt{if}\ b\ \texttt{then}\ \texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od}\ \texttt{fi}$" (which is equivalent to "$S_0;\ \texttt{if}\ b\ \texttt{then}\ \texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od}\ \texttt{else}\ \texttt{skip}\ \texttt{fi}$").

## Answer of exercise 3

1.

$$\dfrac{\dfrac{}{\{P\}\ z := 0\ \{R_2\}}\ (3) \quad \dfrac{\dfrac{}{\{R_2\}\ x_1 := 0\ \{R_1\}}\ (2) \quad \dfrac{}{\{R_1\}\ x_2 := 1\ \{I\}}\ (1)}{\{R_2\}\ x_1 := 0;\ x_2 := 1\ \{I\}}}{\{P\}\ z := 0;\ x_1 := 0;\ x_2 := 1\ \{I\}}$$

où $R_1 = (0 \leq z \leq n \wedge x_1 = \mathit{fib}(z) \wedge 1 = \mathit{fib}(z+1))$ et $R_2 = (0 \leq z \leq n \wedge 0 = \mathit{fib}(z) \wedge 1 = \mathit{fib}(z+1))$.
Justifications:

**(1)** $I[1/x_2] = R_1$

**(2)** $R_1[0/x_1] = R_2$

**(3)** $R_2[0/z] = (0 \geq 0 \wedge 0 \leq n \wedge 0 = \mathit{fib}(0) \wedge 1 = \mathit{fib}(1))$, which is equivalent to $n \geq 0$ since $0 \geq 0$, $0 = \mathit{fib}(0)$, and $1 = \mathit{fib}(1)$ are always true.

2. We must have $I \wedge \neg(z < n) \implies Q$, i.e., $z \geq 0 \wedge z \leq n \wedge x_1 = \mathit{fib}(z) \wedge x_2 = \mathit{fib}(z+1) \wedge z \geq n \implies x_1 = \mathit{fib}(n) \wedge x_2 = \mathit{fib}(n+1)$. It is true, because $z \leq n \wedge z \geq n$ implies $z = n$, which we may combine with $x_1 = \mathit{fib}(z) \wedge x_2 = \mathit{fib}(z+1)$ to obtain $x_1 = \mathit{fib}(n) \wedge x_2 = \mathit{fib}(n+1)$.

3.

$$\begin{aligned}
&wp(z := z+1;\ t := x_1;\ x_1 := x_2;\ x_2 := x_2 + t, I) = \\
&\quad wp(z := z+1;\ t := x_1;\ x_1 := x_2, wp(x_2 := x_2 + t, I)) = \\
&\quad wp(z := z+1;\ t := x_1;\ x_1 := x_2, I[x_2 + t/x_2]) = \\
&\quad wp(z := z+1;\ t := x_1, wp(x_1 := x_2, I[x_2 + t/x_2])) = \\
&\quad wp(z := z+1;\ t := x_1, I[x_2 + t/x_2][x_2/x_1]) = \\
&\quad wp(z := z+1, wp(t := x_1, I[x_2 + t/x_2][x_2/x_1])) = \\
&\quad wp(z := z+1, I[x_2 + t/x_2][x_2/x_1][x_1/t]) = \\
&\quad I[x_2 + t/x_2][x_2/x_1][x_1/t][z+1/z] = \\
&\quad (z \geq 0 \wedge z \leq n \wedge x_1 = \mathit{fib}(z) \wedge x_2 = \mathit{fib}(z+1))[x_2 + t/x_2][x_2/x_1][x_1/t][z+1/z] = \\
&\quad (z \geq 0 \wedge z \leq n \wedge x_1 = \mathit{fib}(z) \wedge x_2 + t = \mathit{fib}(z+1))[x_2/x_1][x_1/t][z+1/z] = \\
&\quad (z \geq 0 \wedge z \leq n \wedge x_2 = \mathit{fib}(z) \wedge x_2 + t = \mathit{fib}(z+1))[x_1/t][z+1/z] = \\
&\quad (z \geq 0 \wedge z \leq n \wedge x_2 = \mathit{fib}(z) \wedge x_2 + x_1 = \mathit{fib}(z+1))[z+1/z] = \\
&\quad (z+1 \geq 0 \wedge z+1 \leq n \wedge x_2 = \mathit{fib}(z+1) \wedge x_2 + x_1 = \mathit{fib}(z+2))
\end{aligned}$$

We must have $I \wedge z < n \Rightarrow 0 \leq z+1 \leq n \wedge x_2 = \mathit{fib}(z+1) \wedge x_2 + x_1 = \mathit{fib}(z+2)$. This implication is valid because $0 \leq z+1$ is the consequence of $0 \leq z$, present in the hypothesis $I$, $z+1 \leq n$ is the consequence of the hypothesis $z < n$, $x_2 = \mathit{fib}(z+1)$ is present in the hypothesis $I$, and $x_2 + x_1 = \mathit{fib}(z+2)$ is the consequence of $0 \leq z \wedge x_1 = \mathit{fib}(z) \wedge x_2 = \mathit{fib}(z+1)$ present in the hypothesis $I$ and of the definition of *fib*: for all $z \geq 0$, $\mathit{fib}(z+2) = \mathit{fib}(z) + \mathit{fib}(z+1)$.

4.

$$\dfrac{T_0 \quad \dfrac{\dfrac{T_1}{\{I\}\ \texttt{while}\ z < n\ \texttt{do}\ z := z+1;\ t := x_1;\ x_1 := x_2;\ x_2 := x_2 + t\ \texttt{od}\ \{I \wedge z \geq n\}}}{\{I\}\ \texttt{while}\ z < n\ \texttt{do}\ z := z+1;\ t := x_1;\ x_1 := x_2;\ x_2 := x_2 + t\ \texttt{od}\ \{Q\}}}{\{P\}\ S_0\ \{Q\}}$$

5. This triple is valid for total correctness because the while loop always termine, with a number of iterations equal to $n$. The expression $n - z$ strictly decreases by one at each iteration. It may never become negative because initially $n - z$ is equal to $n$, which is greater or equal to 0, then when $n - z$ becomes null, the loop condition implies the loop termination.