Univ. Grenoble Alpes | **UFR IM²AG**
MoSIG 1 | **Year 2025-2026**

Programming Language Semantics and Compiler Design
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Midterm Exam of Monday, October 20, 2025

- **Duration**: 1h15.
- Authorized documents up to 3 double-sided sheets of A4 paper.
- Any electronic device is forbidden.
- The grading scale is indicative.
- Exercises are **independent**.
- **The care of your submission will be taken into account.**
- Read each exercise till the end before answering.
- **Indicate your group number on your submission.**
- If you don't know how to answer a question, you may assume the result and proceed with the next question.
- The maximal grade is 20/20.

Exercise 1 — Operational Semantics - applying the rules (5 points)

1. **(3 points)** Give the derivation tree obtained from the following initial configuration, using the natural operational semantics of the **While** language seen in class:

$$(\texttt{while}\ x+y>0\ \texttt{do}\ x:=x+1;\ y:=y-2\ \texttt{od}, [x\mapsto 1, y\mapsto 0])$$

2. **(2 points)** Give the derivation sequence obtained from the same initial configuration, using the structural operational semantics of the **While** language seen in class.

Exercise 2 — Natural Operational Semantics - "do ... while" statement (6 points)

We consider a variant of the **While** language, in which the loop statement "$\texttt{while}\ b\ \texttt{do}\ S_0\ \texttt{od}$" is removed, and replaced by "$\texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od}$", wehere S_0 is a statement and b is a Boolean expression. Informally, this statement has the following behaviour: S_0 is executed, then the condition b is evaluated. If b is true, then the loop is executed once again. Otherwise, it stops.

1. **(3 points)** Complete the following natural operational semantic rules below:

$$\frac{(...,...)\to\sigma'}{(\texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od},\sigma)\to ...}\ \text{if}\ \mathcal{B}[...](...)= ... \qquad \frac{(...,...)\to\sigma' \quad (...,...)\to\sigma''}{(\texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od},\sigma)\to ...}\ \text{if}\ \mathcal{B}[...](...)= ...$$

2. **(3 points)** The structural operational semantics of this statement may be described by a single rule of the following form:

$$\frac{}{(\texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od},\sigma)\Rightarrow(...,\sigma)}$$

Complete this rule by remplacing the "..." symbol by the appropriate statement, belonging to the considered variant of the **While** language. Beware: it is thus forbidden to use the statement "$\texttt{while}\ b\ \texttt{do}\ S_0\ \texttt{od}$", but "$\texttt{do}\ S_0\ \texttt{while}\ b\ \texttt{od}$" and the other statements of the **While** language are authorized.

Exercise 3 — Axiomatic Semantics (9+1 points)

The mathematical expression $\mathit{fib}(n)$ denotes the $n+1$-th element of the Fibonacci sequence, defined by the following three equations, which you may use in your reasoning:

$$\begin{array}{rcll} \mathit{fib}(0) & = & 0 & & (1)\\ \mathit{fib}(1) & = & 1 & & (2)\\ \mathit{fib}(n+2) & = & \mathit{fib}(n)+\mathit{fib}(n+1) & \text{for all}\ n\geq 0 & (3) \end{array}$$

Thus, *fib* defines the sequence 0, 1, 1, 2, 3, 5, 8, 13, 21, etc.

We consider the Hoare triple $\{P\}\ S_0\ \{Q\}$, where :

$$
\begin{array}{rcl}
P & = & (n \geq 0) \\
S_0 & = & S_1;\ \texttt{while}\ z < n\ \texttt{do}\ S_2\ \texttt{od} \\
S_1 & = & z := 0;\ x_1 := 0;\ x_2 := 1 \\
S_2 & = & z := z + 1;\ t := x_1;\ x_1 := x_2;\ x_2 := t + x_1 \\
Q & = & (x_1 = \mathit{fib}(n) \wedge x_2 = \mathit{fib}(n+1))
\end{array}
$$

We propose to show the validity of this triple using logic (including the above mathematical definition of *fib*) and the inference rules seen in class. Care should be taken to justify each of the rules used, by specifying the substitutions and the implications or equivalences that have been applied.
We propose as loop invariant the predicate I defined as follows:

$$I = (0 \leq z \leq n \wedge x_1 = \mathit{fib}(z) \wedge x_2 = \mathit{fib}(z+1))$$

1. **(3 points)** Build a proof tree (which will be named T_0) of the Hoare triple $\{P\}\ S_1\ \{I\}$.
2. **(1 point)** Which logical implication expresses that under the invariant I, the end of the loop guarantees the postcondition Q? Is this implication valid? (Justify.)
3. **(2 points)** We write R for the predicate $wp(S_2, I)$, where wp denotes the weakest precondition calculus seen in class. Compute R.
4. **(1 point)** Which logical implication between $I \wedge z < n$ and R would allow deducing that there exists a proof tree T_1 (we do not ask for T_1) of the Hoare triple $\{I \wedge z < n\}\ S_2\ \{I\}$? Justify that this implication is valid.
5. **(2 points)** Build a proof tree of $\{P\}\ S_0\ \{Q\}$, assuming that the proof trees T_0 and T_1 defined in the previous questions are given.
6. **Bonus (1 point)** Justify, without building the corresponding proof tree, that this Hoare triple is also valid with respect to total correctness.