Licence Sciences et Technologies
Université Joseph Fourier

Mid-term Exam
INF 232: Languages and Automata
L2, 2012/2013

## Reminder about guidelines and some advices/remarks

- Duration: 2 hours (14h00 $\rightarrow$ 16h00).
- No exit before 30 minutes. No entry after 30 minutes.
- Any document of the course or the tutorial is allowed.
- Any electronic device is forbidden (calculator, phone, tablet, etc.).
- Care of your submission will be taken into account. Justify carefully your answers.
- Exercises are independent.
- The exam is graded on 25 points. The grading scale is indicative.

### Exercise 1 (True or False - 3 points)

Answer by True or False to the following questions. Justify *carefully* your answers.

1. Given a language, one can find a deterministic finite-state automaton that recognizes this language.
2. Given a regular language, one can find a deterministic finite-state automaton that recognizes this language.
3. For any regular language, one can find a deterministic finite-state automaton that satisfies the two following conditions:
   - the automaton recognizes this language, and
   - if one removes any state of this automaton, then the obtained automaton recognizes a different language.
4. For any regular language, one can find a finite-state recognizing automaton without any $\epsilon$-transition.

### Exercise 2 (True or False continued - 3 points)

Answer by True or False to the following questions. Justify *carefully* your answers.
Let $\Sigma$ and $\Sigma'$ be two alphabets, let $Q$ be a set of states, and let $\Delta \subseteq Q \times \Sigma \times Q$ be a transition relation. We consider the two automata defined by the following 5-tuples: $A = (Q, q_0, \Sigma, \Delta, F)$ and $A' = (Q, q_0, \Sigma', \Delta, F)$. We note $\mathcal{L}(A)$ and $\mathcal{L}(A')$ the languages recognized by $A$ and $A'$, respectively.

1. If $\Sigma \subseteq \Sigma'$, then $\mathcal{L}(A) \subseteq \mathcal{L}(A')$.
2. $\mathcal{L}(A) = \mathcal{L}(A')$ always holds.
3. If $\Sigma \subset \Sigma'$, then $\mathcal{L}(A) \subset \mathcal{L}(A')$.

### Exercise 3 (Some algorithms - 4 points)

1. Give an algorithm that takes a DFA over an alphabet $\Sigma$ as input and determines whether the language recognized by this automaton is the universal language over $\Sigma$. In this question, the algorithms of the course can be reused without being redefined.

2. Give an algorithm that takes an automaton as input and determines whether this automaton is deterministic. In this question, the algorithms of the course can not be reused.

## Exercise 4 (Regular expressions to automata - 5 points)

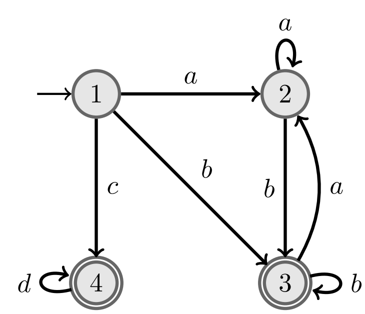
We consider the following regular expression over the alphabet $\{a, b, c, d\}$:

$$\big(a \cdot b + a^* \cdot b + c \cdot d\big) \cdot \big((c \cdot a^* + b \cdot d) + (a \cdot b^* + a \cdot b \cdot d)\big)^*$$

1. Give a non-deterministic automaton with $\epsilon$-transitions that recognizes the language denoted by this regular expression. You are not obliged to follow the compositional method seen in the course. You can perhaps simplify the regular expression before drawing the automaton (with the appropriate justifications).
2. Eliminate the $\epsilon$-transitions in the automaton obtained in the previous question.
3. Determinise the automaton obtained in the previous question.
4. Is this automaton minimal? If the automaton is not minimal, give the minimal automaton.

## Exercise 5 (Automata to regular expressions - 3 points)

We consider the following automaton:




1. Give a regular expression associated to this automaton using the method associating equations to states.

## Exercise 6 (A proof - 3 points)

Let $A_1 = (Q_1, q_0^1, \Sigma, \delta_1, F_1)$ and $A_2 = (Q_2, q_0^2, \Sigma, \delta_2, F_2)$ be two DFA. We consider the $\epsilon$-NFA

$$union(A_1, A_2) = (Q_1 \cup Q_2 \cup \{q_0\}, q_0, \Sigma, \delta, F_1 \cup F_2)$$

with $\delta = \delta_1 \cup \delta_2 \cup \{(q_0, \epsilon, q_0^1), (q_0, \epsilon, q_0^2)\}$.
We note $\mathcal{L}(union(A_1, A_2)), \mathcal{L}(A_1), \mathcal{L}(A_2)$ the languages recognized by $union(A_1, A_2), A_1, A_2$, respectively.

1. For each of the automata A1 and A2 give an explicit example and then build the automaton $union(A1, A2)$, using the construction suggested above.
2. Prove that $\mathcal{L}(union(A_1, A_2)) = \mathcal{L}(A_1) \cup \mathcal{L}(A_2)$.

## Exercise 7 (A non regular language - 4 points)

1. Prove that $\{a^m \cdot b^n \cdot c^p \mid m \geq 0 \text{ et } n > p \geq 0\}$ is not a regular language.