Licence Sciences et Technologies
Université Joseph Fourier

Second session exam
INF 232: Languages and Automata
L2, 2012/2013

Reminder about guidelines and some advices/remarks

– Duration : 2 hours (10 :45am $\rightarrow$ 12 :45pm).
– No exit before 30 minutes. No entry after 30 minutes.
– Any document of the course or the tutorial is allowed.
– Any electronic device is forbidden (calculator, phone, tablet, etc.).
– **Care of your submission will be taken into account.**
– Exercises are independent.
– The grading scale is indicative.

Exercice 1 (True or False - 3 points)

Answer by True or False to the following questions. Justify *carefully* your answers, and without proof.

1. A regular language contains a finite number of words.
2. A complete and deterministic automaton recognizes the universal language.
3. If L is not a finite-state language and and L' is finite then $L \cap L'$ is a finite-state language.
4. The difference between two regular languages is a regular language.
5. Let L be a language, then $L \cap L$ is a finite-state language.
6. In the Floyd verification method, the post-condition has to be implied by at least one condition of the final states.

Exercice 2 (Union of Languages - 2 points)

The union of two non-regular languages L_1 and L_2 can be a regular language or not. Justify without proof your answers to the following questions.

1. Give an example of pair of languages L_1 and L_2 such that $L_1 \cup L_2$ is not regular.
2. Give an example of pair of languages L_1 and L_2 such that $L_1 \cup L_2$ is regular.

Exercice 3 (Regular and non-regular Languages - 5 points)

We consider the alphabet $\Sigma = \{0, 1, 2\}$. Given a word $w \in \Sigma^*$, by $f(w)$ we denote the first position of w which symbol is 1, if such a position exists ; and $|w| + 1$ otherwise. We can inductively define f as follows :
– $f(\epsilon) = 1$.
– $f(1 \cdot u) = 0$, $f(0 \cdot u) = 1 + f(u)$ and $f(2 \cdot u) = 1 + f(u)$.

1. Compute $f(0210)$ and $f(100)$.
2. Given a word $w \in \Sigma^*$, $|w|_0$ denotes the numbers of 0 in w. Let us consider $L = \{w \in \Sigma^* \mid f(w) = |w|_0\}$. Prove that L is not regular.
3. Let M be the language described by the regular expression $(0 + 1)^*$.
   Show that $L \cap M$ is regular.

Exercice 4 (An automaton - 5 points)

We consider the automaton in Figure 1.

1. Suppress ϵ-transitions.
2. Determinize the obtained automaton.
3. Minimize the obtained automaton.

Exercice 5 (Floyd Verification Method - 6 points)

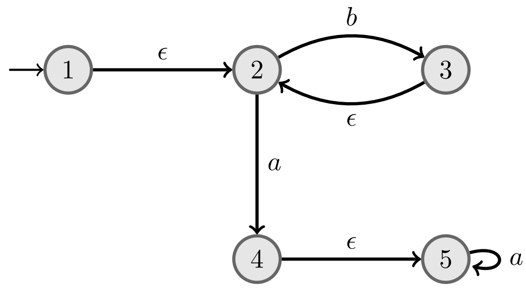

Figure 1: Automaton for exercise 4

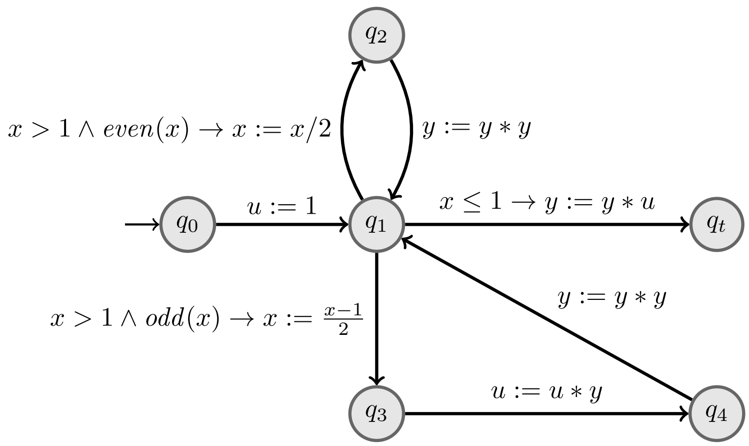

Figure 2: Extended automaton for Exercise 5

We consider the extended automaton in Figure 2.

1. Give the executions of the automaton by considering states σ such that :
   1. $\sigma(x) = 2$, $\sigma(y) = 5$ and $\sigma(u) = 0$,
   2. $\sigma(x) = 3$, $\sigma(y) = 4$ and $\sigma(u) = 0$,

   as initial states.
2. Demonstrate that the extended automaton is partially correct w.r.t. the specification :

$$(x = x_0 \wedge y = y_0 \wedge x_0 > 0, y = y_0^{x_0})$$

   Hint : One can take $P_{q_3} : x \geq 1 \wedge y_0^{x_0} = u * y^{2x+1}$.
   When you will show that the automaton is inductive, you will only consider the following transitions : from q_1 to q_3, from q_3 to q_4, from q_4 to q_1 and from q_1 to q_t.