_	Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Some proof techniques	Maths Reminder
Grenoble Unita	Proofs by contradiction, reducto ad absurdum, contransition	
Alpes	they rely on the principles of propositional and predicate	lasian
Programming Language Semantics and Compiler Design	they rely on the principles of propositional and predicate	logics.
(Sémantique des Langages de Programmation et Compilation)		
Maths Reminders	Proof by structural induction	
	 Proof for the basic elements, atoms, of the set. Proof for composite elements (created by applying) rules: 	
Yliès Falcone	 assume it holds for the immediate components (induction hypothesis 	;)
ylies.falcone@univ-grenoble-alpes.fr — www.ylies.fr Univ. Grenoble Alpes, and LIG-Inria team CORSE	prove the property holds for the composite element	
Master of Sciences in Informatics at Grenoble (MoSIG) Master 1 info	Induction on the shape of a derivation tree	
Univ. Grenoble Alpes - UFR IM ² AG	 Proof for composite trees: 	
www.univ-grenoble-alpes.fr — im2ag.univ-grenoble-alpes.fr	 For each rule R, consider a composite tree where R is the last rule a 	pplied
Academic Year 2020 - 2021	 Proof for the composite tree Assume it holds for subtrees, or premises of the rule (induction hypothes) 	is)
	Proof for the composite tree	
	YWes Falcone (ylies.falcone@univ-grenoble-alpes.fr)	1 1
Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Remind	ers Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC	Maths Reminder
Outline - Maths Reminders	Outline - Maths Reminders	
	Proof by induction	
Proof by induction	Proof by induction	
Proof by structural induction		
	Proof by structural induction	
A notation: derivation tree	A notation: derivation tree	
Yiks Fakcone (ylies.falcone@univ-grenoble-alpes.fr) 2	17 Yliës Falcone (ylies.falcone@univ-grenoble-alpes.fr)	3 1
Univ. Grenoble Alpes. UFR IM ² AG. MoSIG 1 PLCD - Master 1 info SLPC Maths Remind	ers Univ. Grenoble Alpes. UFR IM ² AG. MoSIG 1 PLCD - Master 1 info SLPC	Maths Reminder
Proof by induction	Proof by induction (example)	
Proving a predicate $P(n)$ that depends on some parameter $n \in \mathbb{N}$.		
Example 1 (Predicate)	Example 2 (Proof by induction)	
$P(n) = \text{The sum of the } n \text{ natural numbers is } \frac{n \times (n+1)}{2}.$	Let s prove that $n = n(n+1)$	
$P(n) = "If q \ge 2, we have n \le q".$	$\forall n \in \mathbb{N} : \sum_{i=0}^{n} i = \frac{n(n+2)}{2}$	
r(n) = cvery polynomial of degree n has at most n roots.		
We want to prove $\forall n \in \mathbb{N} \cdot P(n)$	Base case: $\sum_{i=0}^{n} i = 0.$	
	 ► Induction step. ► Suppose that the property holds for some n ∈ N. 	
Principle	► We have:	
Prove the base case: prove that P(0) holds (or P(k) if the minimal value	$\sum_{i=0}^{n+1} i = \sum_{i=0}^{n} i + n + 1$	
of the parameter is k).	$= \frac{1}{2} + n + 1 \text{(induction hypothesis)}$ $= \frac{(n+2)\times(n+1)}{2}$	
Prove the induction step : prove that if for some <i>n</i> , $P(n)$ holds, then $P(n + 1)$ holds	2	
The principle of induction ensures that $P(n)$ holds for any $n > k$		
$r = p \cdot n = p \cdot n = p \cdot n = 0$ and $r = n = n = n = n = n = n = n = n = n = $		
Ville Edward (ed. or. del construction conservable), e la const		
Yiles Faicone (yiles.falcone@univ-grenoble-alpes.fr) 4	1/ YHES Falcone (ylies.falcone@univ-grenoble-alpes.fr)	5 1

$\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^$	2	
Proving a predicate $P(n)$ (but depends on some parameter $n \in N$. That is, we want to prove $\forall n \in N : P(n)$. Principle of complete inductions Prove the base case prove that $P(n)$ holes for $P(n)$ (the minimal value of the base case prove that $P(n)$ holes for $n = 0$. Prove the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of the base case prove that $P(n)$ holes for $n \neq 0$; if the minimal value of tholes for $n \neq 0$; if the minimal val	Univ. Grenoble Alpes, UFR IM ^C AG, MoSIG I PLCD - Master 1 info SLPC Maths Reminders Proof by complete induction	Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Reminders Proof by complete induction: example
Provide a predicate $P(a)$ that depends on some parameter $a \in \mathbb{N}$. Then is, we can to prove for $\mathbb{N} : P(a)$. Prior the base case, prove that $P(b)$ holds (or $P(b)$ if the minimal value of the parameter $h(a)$. Prove the base case, prove that $P(b)$ holds (or $P(b)$ if the minimal value of the parameter $h(a)$. Prove the base case, prove that $P(b)$ holds (or $P(b)$ if the minimal value of the parameter $h(a)$. The prior the base case, prove that $P(b)$ holds for $a_{1}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{1}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{1}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}$. The prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}(a)$. Prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}(a)$. Prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}(a)$. Prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}(a)$. Prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}(a)$. Prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}(a)$. Prior the base case, prove that $P(b)$ holds for $a_{2}(a) \geq b_{2}(a)$. Prior the base case, prove the base case is the case, and the base case is the case, and the base case is the case, and the base case is the ca		The set filesessi sumbar file defined as follows:
Provide a product $P(n)$ that depends on some parameter $n \in \mathbb{N}$. That is, we want to prove $\forall n \in \mathbb{N} : P(n)$. Principle of complete induction Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if for some n . Principle of induction step: prove that if $n = n$ at the some step: n and n		$\mathbf{b}_{n} = 0$
That is, we want to prove the $(A + P(q))$. That is, we want to prove the $(A + P(q))$ holds for $P(q)$ if the minimal value of the parameter is $(A + Q)$. Let us append that the sub-fibrance construction to the prove that if for some n , $\forall w \leq n : P(q)$ holds, the $P(q + 1)$ holds. The prior the base cases prove that if for some n , $\forall w \leq n : P(q)$ holds, the $P(q + 1)$ holds. The prior the base cases prove that if for some n , $\forall w \leq n : P(q)$ holds, the $P(q + 1)$ holds. The prior the base cases prove that if for some n , $\forall w \leq n : P(q)$ holds, the $P(q + 1)$ holds. The prior the base cases prove that if for some n , $\forall w \leq n : P(q)$ holds, for any $n \geq k$. The prior the base cases prove the transmitter is the some that if for some n , $\forall w \leq n : P(q)$ holds, for any $n \geq k$. The prior the base cases base constructions the some $n \geq k$. The prior the base cases base constructions to be explored in the some $n \geq k$. The prior the base constructions the some $n \geq k$ is a set of the distance of the some $n \geq k$. The some $n \geq 0$ denotes the some $n \geq k$ is a set of the distance of	Proving a predicate $P(n)$ that depends on some parameter $n \in \mathbb{N}$.	$f_{1} = 0$
The fit, we want to prove the $k : P(q)$. Principle of complete induction Principle of complete induction Principle of complete induction Principle of complete induction atter, prove that if not fit is a multiple of 3. Prove the base case; prove that $P(q)$ holds for $r(q)$ if the minimal value of the parameter is A_1 . Prove the base case; prove that $P(q)$ holds for $r(q)$ if if the minimal value of the parameter is A_1 . Prove the base case; prove that $P(q)$ holds for $r(q)$ if if the minimal value of the parameter is A_1 . Prove the complete induction step: prove that if not fill for any $n \ge \lambda$. Prove the complete induction step: prove that if not fill for any $n \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the complete induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q)$ holds for $r(q) \ge \lambda$. Prove the induction is $P(q) = P(q)$ holds for $P(q) = P(q)$ holds for $P(q) = P(q)$. Prove the induction is $P(q) = P(q)$ holds for $P(q) = P(q)$ holds fore $P(q) = P(q)$ holds for $P(q) = P(q)$		$\forall k > 2: f_{k} = f_{k-1} + f_{k-2}$
Proceeding of complete inductionProve the function of the P(n) holds for $P(k)$ if the minimal value of the complete induction step, prove that $P(n)$ holds for $P(k)$ if the minimal value of $P(n)$ holds for $P(n)$	That is, we want to prove $orall n \in \mathbb{N}: P(n).$	Let us prove that the n th Eikensesi number is even iff n is a multiple of 2
Proof be besided prove that $P(0)$ holds (or $P(k)$ if the minimal value of the parameter is k). Prove the second prove that $P(0)$ holds (or $P(k)$ if the minimal value of the parameter is k). Prove the complete induction step, prove that if for some n , $\forall n \leq n \leq 1$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of induction ensures that $P(n)$ holds for any $n \geq k$. The principle of principle of the prin the principle of the principle of the principle o		Let us prove that the <i>n</i> -th Fibonacci number is even in <i>n</i> is a multiple of 3.
Prove the base case grows that $P(0)$ holds (or $P(k)$) the minimal value of the prove the computer starts $P(0)$ holds for any $p \ge k$. Prove the computer starts makes $P(n + 1)$ holds. The principle of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. We for example of induction ensures that $P(0)$ holds for any $p \ge k$. Proof by distances Proof by distances Proof by distances Proof by distances the ensures Proof by distances the example of $P(0)$ holds for any $p \ge k$. We for example of $P(0)$ holds for $P(0)$ holds for any $p \ge k$. We for example of $P(0)$ holds for $P(0)$ holds for any $p \ge k$. Proof by distances the example of $P(0)$ holds for any $p \ge k$. We for example $P(0)$ holds for $P(0)$ holds for any $p \ge k$. We for example $P(0)$ holds for $P(0)$ holds for any $p \ge k$. We for example $P(0)$ holds for $P(0)$ holds for any $p \ge k$. Proof by distances the example $P(0)$ holds for $P(0)$ holds for any $p \ge k$. We for example $P(0)$ holds for $P(0)$ holds for $P(0)$ holds for any $p \ge k$. Proof by distances the example $P(0)$ holds for any $P(0)$ holds for any P	Principle of complete induction	Example 3 (The <i>n</i> -th Fibonacci number is even iff <i>n</i> is a multiple of 3.)
$\frac{1}{2} + \frac{1}{2} \cos \theta = \cos \theta = \sin \theta $	Prove the base case: prove that P(0) holds (or P(k) if the minimal value of the parameter is k)	Base case. We can see that it holds for $n = 0$.
$ \begin{array}{c} \forall n \leq n : (P(n) holds, then P(n + 1) holds. \\ \hline The principle of induction ensures that P(n) holds for any n \geq k. \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	 Prove the complete induction step: prove that if for some n, 	 Let us suppose that the property holds for any integer lesser than or equal
The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $n \ge k$. The principle of induction ensures that $P(n)$ holds for any $P(n)$ holds for any $P(n) = (P(n) = N \cap A_1, R \in B(R))$. Example 3 (finary trees) if each numbers) For an ensure $n \ge N \to k$. The principle of index $P(n) = (Ens P(n) \cup E(N \cap A_1, R \in B(R)))$. Example 3 (finary trees) if each numbers) But $P(n) = (Ens P(n) \cup N \cap A_1, R \in B($	$\forall m \leq n : P(m)$ holds, then $P(n+1)$ holds.	to some $n \in \mathbb{N}$. Consider f_{n+1} and distinguish three cases according to the rest of the
$\frac{1}{20} \frac{1}{20} \frac$	The principle of induction ensures that $P(n)$ holds for any $n \ge k$.	division of $n+1$ by 3.
We transform functions introduced particulated as a subject of the subject indication product inter the subject indication of the s		► Case $n + 1 \mod 3 = 0$. $s_{3k} = s_{3k-1} + s_{3k-2} = odd + odd = even$ ► Case $n + 1 \mod 3 = 1$. $s_{3k+1} = s_{3k} + s_{3k-1} = even + odd = odd$ ► Case $n + 1 \mod 3 = 2$. $s_{3k+2} = s_{3k+1} + s_{5k} = odd + even = odd$
With developmentation provided in the formation provector preverepresented in the formation provided in the formatio		
Inter-Section Assumption Section Proof by structural inductionInter-Section Assumption Section Proof by structural inductionInter-Section Assumption Section Proof by structural inductionProof by structural inductionA metation: demotion treeInter-Section Assumption Section Proof by structural inductionDefinition 4 (closure)A metation: demotion treeInter-Section Proof by structural inductionDefinition 5 (Construction rule)A construction rule for a set states either: Inter-Section Proof by structural inductionA metation: demotion treeInter-Section Proof by structural inductionA metation: demotion treeInter-Section Proof by structural inductionA metation: demotion treeInter-Section Proof by structural inductionInter-Section Proof by structural inductionInter-Section Proof by structural inductionInter-Section Proof by structural inductionInter-Section Proof by structural induction for a set states either: Proof by structural induction definitionInter-Section Proof by structural induction provide a new definitionInter-Section Proof by structural inductive definition on E is a family of rules defining the smallest subset of E that is closed by these rules.Inter-Section Proof by Section Proof by Se	Yliës Falcone (ylies.falcone@univ-grenoble-alpes.fr) 6 [17	YBies Falcone (ylies.falcone@univ-grenoble-alpes.fr) 7
Proof by inductionLet us consider: $F a set.$ $A = a st.$ $A = a st.$ $A = b a b a b a b a b b a b$	Univ. Grenoble Alpes, UFR Mr ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Reminders Outline - Maths Reminders	Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Reminder Inductive/Compositional definitions
Pred by indiction $A = a \text{ set}$, $A = f a \text{ set}$, $A =$		Let us consider:
Proof by induction Proof by induction Proof by structural induction A notation: definition A closure Proof by structural induction A notation: definition A closure Proof by structural induction A notation: definition A closure Definition 5 (Construction rule) A construction rule for a set states either • that a basis element belongs to the set, or • thou to produce a new element from existing elements (production rule given by a partial function). Definition 6 (Line definition) A notation: definition at E is a finally of rules defining the smallest subset of E that is closed by these rules. • those contact when the set is a finally of rules defining the smallest subset of E that is closed by these rules. • those contact when the set is a finally of rules defining the smallest subset of E that is closed by these rules. • those contact when the set is a finally of rules defining the smallest subset of E that is closed by these rules. • those contact when the set is a finally of rules defining the smallest subset of E that is closed by these rules. • those contact when the set is a finally of rules definition) A rule is a hinary tree is a		► E a set ,
Proof by inductionProof by structural inductionA notation: deviation interesA notation: deviation: deviation		▶ $f: E \times E \times \times E \to E$ a partial function,
Proof by inductionDefinition 4 (closure)A is closed by firt $((A \times \times A) \subseteq A.$ A notation: derivation treeA notation: derivation treeA notation: derivation treeA notation: derivation treeA notation: derivation rule for a set state site:- how to produce a new element from existing elements (production rule given by a partial function).An inductive definition of E is a family of rules defining the smallest subset of E that a basis element beings to the set, or- how to produce a new element from existing elements (production rule given by a partial function).An inductive definition of E is a family of rules defining the smallest subset of E that is closed by these rules.More come derive definitions: examplesInductive definitions: examplesExample 7 (Natural numbers) How can define them?> basis element 0 > 1 rule $x \mapsto x = x(x)$ 2 is the natural numbers)> basis element 0 > 1 rule $x \mapsto x = x(x)$ Example 8 (Even numbers) > basis element 0 > 1 rule $x \mapsto x + 2$ Example 9 (Palindromes on $\{a, b\}$) > basis element 0 > 1 rule $x \mapsto x = x$ Example 9 (Palindromes on $\{a, b\}$) > basis element (a, a, b) > 2 rules: $w \mapsto a w = a, w \mapsto b w = b$ Multiple 10 (Binary trees) $(a, f, f) e \in \mathbb{N} \land t, e \in Bt(\mathbb{N})$ Example 9 (Palindromes on $\{a, b\}$) > basis element (a, a, b) > 2 rules: $w \mapsto a w = a, w \mapsto b w = b$ Multiple 10 (Binary trees) $(a, f, f) e \in \mathbb{N} \land t, e \in Bt(\mathbb{N})$ Multiple 10 (Binary trees) $(a, f, f) e \in \mathbb{N} \land t, e \in Bt(\mathbb{N})$ Multiple 10 (Binary trees) $(a, f, f) e \in \mathbb{N} \land t, e \in Bt(\mathbb$		• $A \subseteq E$ a subset of E .
Proof by structural induction A notation: derivation tree A notation: derivation: derivation A notation: derivation: derivation A notation: derivation: derivation A notation: derivation: d	Proof by induction	Definition 4 (closure)
Private of production numbers of the stands of the stan	Proof by structural induction	A is closed by f iff $f(A \times \ldots \times A) \subseteq A$.
A notation: derivation tree A notation: derivation tree A construction of Construction (Construction of Construction of Con		Definition 5 (Construction rule)
$ \text{We Falses (cites characteristics)} \\ \text{We false (cites characteristics)} \\ We false (cites characteri$	A notation: derivation tree	A construction rule for a set states either:
$ \begin{array}{c} \begin{tabular}{lllllllllllllllllllllllllllllllllll$		that a basis element belongs to the set, or
f(x) =		how to produce a new element from existing elements (production rule
Uttle face (Interfacements regently expected spectral)Definition 6 (Inductive definition) An inductive definition on E is a family of rules defining the smallest subset of E that is closed by these rules.Vite face (Interfacements regently expected spectral)Vite facements regently expected spectral)Vite face (Interfacements regently expected spectral)Vite facements regently expected spectral)Vite facements regently expected spectral)Vite face (Interfacements regently expected spectral)Vite facements regently expected spectral)Vite facements regently expected spectral)Vite face (Interfacements regently expected spectral)Vite facements regently expected spectral)Vite facements regently expected spectral)Vite face (Interfacements regently expected spectral)Vite facements regently expected spectral) <td< td=""><td></td><td>given by a partial function).</td></td<>		given by a partial function).
$ \begin{array}{c} \text{An inductive definition on E is a family of rules defining the smallest subset of E that is doed by these rules. \\ \hline \\ E \text{ that is doed by these rules.} \\ \hline \\ \hline \\ \text{Une. General Age, URR M2Ac, Model 1 RCO- Mater 1 left SUPC \\ \hline \\ \text{Inductive definitions: examples} \\ \hline \\ \hline \\ \text{Inductive definitions: examples} \\ \hline \\ \text{Adets Remote Information on E is a family of rules defining the smallest subset of E that is doed by these rules. \\ \hline \\ \hline \\ \text{Une. General Age, URR M2Ac, Model 1 RCO- Mater 1 left SUPC \\ \hline \\ \text{Inductive definitions: examples} \\ \hline \\ \text{Example 7 (Natural numbers)} \\ \text{How can define them?} \\ \hline \\ \text{b asis element 0} \\ \hline \\ \text{b asis selement 0} \\ \hline \\ \text{b asis element s (a, a, b)} \\ \hline \\ \text{b asis element s (a, a, b)} \\ \hline \\ \text{b asis element s (a, a, b)} \\ \hline \\ \text{b asis element s (a, a, b)} \\ \hline \\ \text{b asis element s (a, a, b)} \\ \hline \\ \text{b asis element s (a, a, b)} \\ \hline \\ b asis element s (a, a, b) \\ \hline \\ \text{b asis element s (a, a, b) \\ \hline \\ \text{b asis element s (a, a, b) \\ \hline \\ \text{b asis element s (a, a, b) \\ \hline \\ \text{b asis element s (a, a, b) \\ \hline \\ \text{b asis element s (a, a, b) \\ \hline \\ \text{b asis element s (a, a, b) \\ \hline \\ \text{b asis element s (a, a, b) \\ \hline \\ \text{b asis element s (a, b, b) $		Definition 6 (Inductive definition)
Vite Forme (size, falcombasty granitive signs, fr)If is Concerning of the falcombasty granitive signs, from the falcombasty granitive sinters, from		An inductive definition on <i>E</i> is a family of rules defining the smallest subset of
Vite Falsen (title: falsendedir: grandle: types () (1) Vite Falsen (1) (1) (1) Example 7 (Natural numbers) (1) (1) Auss Remoter (1) (1) (1) Auss Remoter (1) (1) (1) (1) Example 8 (Even numbers) (1) (1) (1) (1) (1) Example 9 (Palindromes on {a, b}) (1) (1) (1) (1) (1) (1) Solution (1) (1) (1) (1) (1) (1) (1) (1) Vite Falsen (title falsendire ()		E that is closed by these fulles.
$\begin{aligned} & \text{Line definitions: examples} \\ & \text{Lexample 7 (Natural numbers)} \\ & \text{How can define them?} \\ & \text{basis element 0} \\ & \text{b rule: } x \mapsto succ(x) \\ & 2 \text{ is the natural numbers} \\ & \text{b asis element 0} \\ & \text{b rule: } x \mapsto succ(x) \\ & 2 \text{ is the natural numbers} \\ & \text{b asis element 0} \\ & \text{b rule: } x \mapsto succ(x) \\ & 2 \text{ is the natural numbers} \\ & \text{b asis element 0} \\ & \text{b rule: } x \mapsto succ(x) \\ & \text{b asis element 0} \\ & \text{b rule: } x \mapsto succ(x) \\ & \text{b asis element 0} \\ & \text{b rule: } x \mapsto x + 2 \\ \\ & \text{Example 8 (Even numbers)} \\ & \text{b asis element 0} \\ & \text{b rule: } x \mapsto x + 2 \\ \\ & \text{Example 9 (Palindromes on \{a, b\}) \\ & \text{b asis elements } \epsilon, a, b \\ & \text{b 2 rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b \end{aligned}$	Yliës Falcone (ylies.falcone@univ-gremoble-alpes.fr) 8 17	YWes Falcone (ylies.falcone@univ-gremoble-alpes.fr) 9 [1
Example 7 (Natural numbers) How can define them? basis element 0 1 rule: $x \mapsto succ(x)$ 2 is the natural number defined as $succ(succ(0))$ Example 8 (Even numbers) basis element 0 1 rule $x \mapsto x + 2$ Example 9 (Palindromes on {a, b}) basis elements ϵ, a, b 2 rules: $w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ With Element (rule a discustory regarding regar	Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Reminders Inductive definitions: examples	Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Reminder Binary trees Definition and examples
Example 7 (Natural numbers) How can define them? basis element 0 1 rule: $x \mapsto succ(x)$ 2 is the natural number defined as $succ(succ(0))$ Example 8 (Even numbers) basis element 0 1 rule $x \mapsto x + 2$ Example 9 (Palindromes on $\{a, b\}$) basis elements ϵ, a, b basis elements ϵ, a, b comparison elements ϵ, a, b comparison elements ϵ, a, b basis elements ϵ, a, b comparison		Definition 10 (Binary Tree – Informal definition)
How can define them? • basis element 0 • 1 rule: $x \mapsto succ(x)$ 2 is the natural number defined as $succ(succ(0))$ Example 8 (Even numbers) • basis element 0 • 1 rule $x \mapsto x + 2$ Example 9 (Palindromes on $\{a, b\}$) • basis elements ϵ, a, b • 2 rules: $w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ Will Patements (subscription for the subscription f	Example 7 (Natural numbers)	A tree is a binary tree if each node has <i>at most two children</i> (possibly empty).
▶ basis element 0 ▶ 1 rule: $x \mapsto succ(x)$ 2 is the natural number defined as $succ(succ(0))$ Example 8 (Even numbers) ▶ basis element 0 ▶ 1 rule $x \mapsto x + 2$ Example 9 (Palindromes on {a, b}) ▶ basis elements c, a, b ▶ 2 rules: $w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ With Edoted (vitue, falloged pattern	How can define them?	Definition 11 (Pinney Tree Mathematical definition)
For the strategy set $D(Lt)$ s.t. 2 is the natural number defined as $succ(succ(0))$ Example 8 (Even numbers) b basis element 0 1 rule x ↦ x + 2 Example 9 (Palindromes on {a, b}) b basis elements ϵ , a , b 2 rules: w ↦ a · w · a, w ↦ b · w · b With Edeme (class falcendary graphic-place (r)) With Edeme (class falcendary graphic-place (r)) With Edeme (class falcendary graphic-place (r))	► basis element 0	Deminition 11 (Binary Tree – Mathematical definition) The smallest set Bt(Elt) s + ·
$b_{2} \text{ is the natural number defined as } succ(succ(0))$ $b_{2} \text{ is the natural numbers})$ $b_{2} \text{ is the selements } \epsilon, a, b$ $b_{2} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{2} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto a \cdot w \cdot b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto b \cdot w \cdot b$ $b_{3} \text{ rules: } w \mapsto b \cdot w \cdot b$	▶ 1 rule: $x \mapsto succ(x)$	
Example 8 (Even numbers) $ basis element 0 basis element 0 basis element 0 basis elements \epsilon, a, b basis elements \epsilon, a, b basis elements \epsilon, a, b basis elements \epsilon, a, b basis elements \epsilon, a, b basis elements e, a, b control to the elements elements elements e control to the elements elements elements elements e control to the elements ele$	2 is the natural number defined as <i>succ</i> (<i>succ</i> (0))	$Bt(Elt) = \{EmptyT\} \cup \{Node(tL, e, tR) \mid e \in Elt \land tL, tR \in Bt(Elt)\}$
▶ basis element 0 ▶ 1 rule x ↦ x + 2 Example 9 (Palindromes on {a, b}) ▶ basis elements ε, a, b ▶ 2 rules: w ↦ a ⋅ w ⋅ a, w ↦ b ⋅ w ⋅ b With Edome (rules, falcensebury granuble raless, ft)	Example 8 (Even numbers)	Example 12 (Binary trees of natural numbers)
$ 1 \text{ rule } x \mapsto x + 2 $ Example 9 (Palindromes on {a, b}) $ b \text{ basis elements } \epsilon, a, b $ $ 2 \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b $ $ 2 \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b $ $ 2 \text{ rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b $ $ 2 \text{ rules: } (128 \text{ followeduality granuble allows fr}) $ $ 2 \text{ rules: followeduality granuble allows fr} $	► basis element 0	$ Bt(\mathbb{N}) = \{EmptyT\} \cup \{Node(tL, e, tR) \mid e \in \mathbb{N} \land tL, tR \in Bt(\mathbb{N})\}$
Example 9 (Palindromes on $\{a, b\}$) basis elements ϵ, a, b basis elements ϵ, a, b constraints 100 constraints 100 constrain	▶ 1 rule $x \mapsto x + 2$	
Example 9 (Palindromes on $\{a, b\}$) \blacktriangleright basis elements ϵ, a, b \triangleright 2 rules: $w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$ Vibs Falcone (rules, falconedumity granoble alloss, fr) 1017 Vibs Falcone (rules, falconedumity granoble alloss, fr)		Example 13 (Binary trees)
$ \begin{array}{c} \flat \text{ basis elements } \epsilon, a, b \\ \flat \text{ 2 rules: } w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b \\ \end{array} \\ \end{array} \\ \begin{array}{c} 30 & 74 & \text{'s'} & \text{'a'} & \text{'mammal'' "insect''} \\ \hline 70 & 12 & 8 & 7 & \text{'l'} & \text{'dog'' "cow'' "bee'' "fly''} \\ \end{array} \\ \end{array} \\ \begin{array}{c} 1017 & Vies Falcons (vies, falconschartive grandule views falconschartive views falconschartive grandule views falconschartive grandule views falconschartive grandule views falconschartive grandule views falconschartive views falconschartive grandule views falconschartive views falconschartive grandule views falconschartive vi$	Example 9 (Palindromes on $\{a, b\}$)	100 'd' "animal" / \ / \ / \
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	• basis elements ϵ, a, b	
Yliks Falcone (vlies. falcone@univ-greenoble=aloes.fr) 10177 Yliks Falcone (vlies. falcone@univ-greenoble=aloes.fr) / / / / / / / / / / / / / / / / / / /	▶ 2 rules: $w \mapsto a \cdot w \cdot a, w \mapsto b \cdot w \cdot b$	30 74 's' 'a' "mammal" "insect"
Yiks Falcone (vise, falcone@univ-grenoble-aloes, fr) 10177 Viks Falcone (vise, falcone@univ-grenoble-aloes fr) 10177		
THE FRENCH FULL AND A STREAM AND A ST	Viik Estone (u) (as falconefus (useranoblas) has fu)	/ U 12 O / I UOg COW Dee IIY

Univ. Grenoble Apes, UFR IM ² AC, MoSiG 1 PLCD - Master 1 info SLPC Proof by Structural Induction	Maths Reminders	Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Reminders Proof by Structural Induction: example
Proving that the proof holds for any element "however it is built".		Example 14 (Proofs by induction) All proofs by induction are proofs by structural induction where the inductive set is \mathbb{N} .
Principle		
 Proof for the basic elements, atoms, of the set. Proof for composite elements (created by applying) rules: assume it holds for the immediate components (induction hypothesis) prove the property holds for the composite element 		 Example 15 (Properties of size and depth of a binary tree) Let us consider t ∈ Bt(Elt), a binary tree: depth(t) be the depth of tree t: length of longest path from root to leaf. size(t) be the size of tree t: number of nodes + leaves. For any type Elt and any t ∈ Bt(Elt): depth(t) ≤ size(t), size(t) ≤ 2^{depth(t)-1}.
Yliës Fakone (ylles.falcone@univ-grenoble-alpes.fr)	12 17	Yliks Falcone (ylies.falcone@univ-grenoble-alpes.fr) 13 [17
Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Inductive definitions: examples	Maths Reminders	Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Reminders Outline - Maths Reminders
Example 16 (Natural numbers) How can define them? ► basis element 0 ► 1 rule: x → succ(x) 2 is the natural number defined as succ(succ(0))		Proof by induction
Example 17 (Even numbers)		Proof by structural induction
hasis element: 0:		A notation: derivation tree
1 rule: $x \mapsto x + 2$.		
Example 18 (Palindromes on $\{a, b\}$)		
(a, b)		
► Dasis elements: ϵ, a, b, b ► 2 rules: $w \mapsto a \cdot w \cdot a w \mapsto b \cdot w \cdot b$		
Yliès Falcone (ylies.falcone@univ-grenoble-alpes.fr)	14 17	YHes Falcone (ylies.falcone@univ-grenoble-alpes.fr) 15 17
Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC A notation: derivation tree	Maths Reminders	Univ. Grenoble Alpes, UFR IM ² AG, MoSIG 1 PLCD - Master 1 info SLPC Maths Reminders Abstract syntax trees and derivation trees
		Consider an abstract syntax tree, produced by syntactic analysis.
Notation for $t = f(x_1, \ldots, x_n)$		Derivation tree
$\frac{x_1 \dots x_n}{t} f$		For each node, computing information from the information of its sons.
"t is built/obtained from x_1, \ldots, x_n " by applying operator f .		Example 20 (Derivation trees in type analysis)
Example 10 (Derivation troos)		we obtain for each node a type (or error) based on types of its sons.
= cuce(cuce(cuce(0))) is a patient number		Definition of derivation trees by a formal system
$\frac{\overline{0}}{1}$		Generally, we have information, stored in some environment Γ . The formal system states how to <i>deduce</i> knowledge from existing knowledge, where knowledge is of the form $\Gamma \vdash \mathcal{P}$, which means \mathcal{P} holds on Γ .
$\overline{2}^{\text{succ}}$		a set of axiom schemes
 aba is a palindrome: ababa is a palindrome: 		a set of inference rules : a rule of the form
\overline{b} $\overline{\overline{a}}$		$\frac{\Gamma_1 \vdash \mathcal{P}_1 \cdots \Gamma_n \vdash \mathcal{P}_n}{\Gamma \vdash \mathcal{C}}$
aba <u>bab</u> aba		"if the hypothesis (premisses) \mathcal{P}_i hold, then the conclusion $\mathcal C$ holds.
Ylies Falcone (ylies.falcone@univ-grenoble-alpes.fr)	16 17	Yiks Falcone (ylies.falcone@univ-grenoble-alpes.fr) 17 [17