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# Software security, secure programming

## Lecture 5: Static Analysis (in a nutshell)

Master M2 Cybersecurity

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*statically compute some information about (an approximation of) the program behavior*

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## Possible work-arounds:

- over-approximate the pgm behaviour
  → result is sound (no false negatives), but incomplete (∃ false positives)
- under-approximate the pgm behaviour
  → result is complete (no false negatives), but unsound (∃ false negative)
- non-terminating analysis
  → **if** the analysis terminates, **then** the result is sound and complete

# What static analysis can be used for ?

## General applications

- compiler optimization
  e.g., active variables, available expressions, constant propagations, etc.
- program verification
  e.g., invariant, post-conditions, etc.
- worst-case execution time computation
- parallelization
- etc.

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## In the "software security" context

- disassembling
  e.g., what are the targets of a dynamic jump
  (`be eax`, content of `eax` ?)
- error and vulnerability detection
  memory error (Null-pointer dereference, out-of-bound array access), use-after-free, arithmetic overflow, etc.
- information-flow analysis (integrity, confidentiality, taint analysis)
- "semantic pattern" recognition
- etc.

# Outline

- Overview
- Principles
- Weakest Preconditions
- Abstract Interpretation
- Value-Set Analysis (VSA)
- Conclusion

# How to proceed ?

## Typical problems

- need to reason on a set of executions (not on a single one)

  ex: `x = y * z`

  → compute values of `x` for all possible values of `y` and `z` ?

- need to cope with loops

  ex: `while (x < y) do ... end`

  → infer the loop behavior for all possible values of `x` and `y` ?

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## A solution: over-approximate the program behavior

1. propagate an abstract state (over approximating the memory content)

   e.g., $x > 0$, $p \neq NULL$, $x \leq y + z$, $p$ and $q$ are aliases, etc.

   → depends on the properties you want to check !
2. **safely** merge memory abstract states produced from ≠ paths
3. make loop iterations always finite

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**Pb:** How to find a suitable abstract domains ?
→ accuracy vs scalability trade-offs . . .

# Outline

Overview

Principles

Weakest Preconditions

Abstract Interpretation

Value-Set Analysis (VSA)

Conclusion

# A basic programming language

## Syntax

$$
\begin{array}{lcl}
\text{Exp} & ::= & x \mid n \mid \text{op}\,(\text{Exp}, \ldots \text{Exp}) \\
\text{Stm} & ::= & x := \text{Exp} \\
 & ::= & \text{Stm}\ ;\ \text{Stm} \\
 & ::= & \text{skip} \\
 & ::= & \text{if}\ \ \text{Exp}\ \ \text{then}\ \ \text{Stm}\ \ \text{else}\ \ \text{Stm} \\
 & ::= & \text{while}\ \ \text{Exp}\ \ \text{do}\ \ \text{Stm}\ \ \text{end} \\
 & ::= & \text{assert}\ \ \text{Exp}
\end{array}
$$

In practice : arrays, structures, pointers, procedures, etc.

# Axiomatic Semantics

$\Rightarrow$ programs viewed as predicate transformers where predicates are assertions on program variables (Hoare, Dijkstra 1976).

- Weakest Preconditions (*wp*) : backward computation
  Example :

$$x \geq 0 \ \{x := x + 1;\} \ x > 0$$

- Strongest Postcondition (*sp*) : forward computation
  Example :

$$x \geq 0 \ \{x := x + 1;\} \ x > 0$$

# Weakest precondition / Strongest postcondition

Let $I$ a statement, $P$, $R$, $'$, $R'$ some predicats

The weakest precondition $P = wp(I, R)$ is such that:

$$\forall P' \ (P' \Rightarrow wp(I, R)) \Rightarrow (P' \Rightarrow P)$$

A precondition $P'$ stronger than $x \geq 0 : x > 5$.

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A precondition $P'$ stronger than $x \geq 0$ : $x > 5$.

The strongest postcondition $R = sp(R, I)$ is such that:

$$\forall R' \ (sp(P, I) \Rightarrow R' \Rightarrow (R \Rightarrow R')$$

A postcondition $R'$ weaker than $x \geq 0$ : $x > -2$.

# Substitution - free/bounded variables

## Free and bounded variables

A variable $x$ is bounded (resp. free) within formula $F$ iff $F$ contains an occurrence of $x$ which is (resp. which is not) within the scope of a quantifier.

**Example:**

$$\varphi \equiv P(y, x) \wedge \ \forall x \ . \ Q(x, y)$$

$\hookrightarrow$ there is both a free and a bounded occurrence of $x$ in $\varphi$

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**Example:**

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## Substitution

$P[E/x]$ is the formula $P$ in which all free occurrences of variable $x$ have been replaced by the term $E$.

**Example:**

$$(\varphi[x+1/x])[f/y] \equiv P(f, x+1) \wedge \ \forall x \ . \ Q(x, f)$$

# Computing weakest preconditions: basic instructions

| **Statement** | *def.* | **WP** |
|---|---|---|
| $wp(\text{skip}, R)$ | $\hat{=}$ | $R$ |
| $wp(x := e, R)$ | $\hat{=}$ | $R[e/x]$ |
| $wp(i_1 \; ; \; i_2, R)$ | $\hat{=}$ | $wp(i_1, wp(i_2, R))$ |
| $wp(\text{assert}(e), R)$ | $\hat{=}$ | $e \wedge \; R$ |

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Examples:

1. $wp(x := x + 1, x > 0)$
2. $wp(z := 2\ ;\ y := z + 1\ ;\ x := z + y,\ x \in 3..8)$

# Another way to write WPs

$R$
**skip**;

$R[e/x]$
$\mathbf{x} := \mathbf{e}$;

$wp(i_1, wp(i_2, R))$
$\mathbf{i_1}$;
$wp(i_2, R)$
$\mathbf{i_2}$;

$P \wedge R$
assert(**P**)

# Example

$2 + 2 + 1 \in 3..8$

**z:=2 ;**

$z + z + 1 \in 3..8$

**y:=z+1 ;**

$z + y \in 3..8$

**x:=z+y;**

$x \in 3..8$

# Computing weakest precondition: conditional statement

$$\boxed{\begin{array}{c} wp(\text{if } P \text{ then } i_1 \text{else } i_2 \text{ end}, R) \\ \hat{=}\ (P \Rightarrow wp(i_1, R)) \land\ (\neg P \Rightarrow wp(i_2, R)) \end{array}}$$

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Examples:

- Define $wp(\text{if } e \text{ then } i \text{ end }, R)$.

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Examples:

- Define $wp(\text{if } e \text{ then } i \text{ end }, R)$.
- What does the following program compute ? Prove the result . . .

```
begin
  if $x > y$ then $m := x$ else $m := y$ end ;
  if $z > m$ then $m := z$ end
end
```

# Solution (1)

```
$(x > y \Rightarrow F_1[x/m]) \wedge (\neg(x > y) \Rightarrow]F_1[y/m])$      $= F_2$
if $x > y$
  $F_1[x/m]$
  then $m := x$
  $F_1[y/m]$
  else $m := y$ end ;
$(z > m \Rightarrow R_1[z/m]) \wedge (\neg(z > m) \Rightarrow R_1)$      $= F_1$
if $z > m$
   $R_1[z/m]$ ;
  then $m := z$
   $R_1$ ;
  else skip ;
end
$R_1$
```

# Solution (2)

Postcondition :

$$(m = x \vee \ m = y \vee \ m = z) \wedge \ m \geq \ x \wedge \ m \geq \ y \wedge \ m \geq \ z$$

Let's process $R_1 = m \geq x$.

**Computing $F_1$ :**

$$(z > m \Rightarrow m[z/m] \geq x) \wedge \ (\neg(z > m) \Rightarrow m \geq x)$$

**which can be rewritten:**

$$(z > m \Rightarrow z \geq x) \wedge (\neg(z > m) \Rightarrow m \geq x)$$

# Solution (3)

Computing $F_2$:

$$(x > y \ \Rightarrow\ F_1[x/m]) \wedge (\neg(x > y) \ \Rightarrow\ F_1[y/m])$$

leading to:

$$\begin{array}{lll} (x > y \wedge\ z > x & \Rightarrow z \geq x) & \wedge \\ (x > y \wedge\ \neg(z > x) & \Rightarrow x \geq x) & \wedge \\ (\neg(x > y) \wedge\ z > y & \Rightarrow x \geq x) & \wedge \\ (\neg(x > y) \wedge\ \neg(z > y) & \Rightarrow y \geq x) & \end{array}$$

Each of these 4 propositions is equivalent to **true**.

# Computing weakest precondition: iteration

$$wp(\text{while } b \text{ do } S \text{ end }, R) \quad ?$$

## Partial correctness

→ compute the WP **assuming the loop will terminate**

- need to reason about an arbitrary number of iteration;
- find a loop invariant $I$ such that:
  1. $I$ is preserved by the loop body:
     $$I \wedge b \Rightarrow wp(S, I)$$
  2. if and when the loop terminates, the post-condition holds:
     $$I \wedge \neg b \Rightarrow R$$

**Then**

$$wp(\text{while } b \text{ do } S \text{ end }, R) = I$$

# Computing weakest precondition: iteration

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## Partial correctness

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**Then**

$$wp(\text{while } \ b \text{ do } \ S \text{ end }, R) = I$$

Total correctness: prove that the loop **do** terminate . . .
need to introduce a loop variant
(i.e, a measure strictly decreasing at each iteration towards a limit).

# Example

## Prove the following program using WP

```
{x=n && n>0}
  y := 1 ;
  while x <> 1 do
     y := y*x ;
     x := x-1 ;
   end
{y=n! && n>0}
```

# Implementing WP computation ?

1. WP computation:
   - based on the program structure (Abstract Syntax Tree)

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1. WP computation:
   - based on the program structure (Abstract Syntax Tree)
   - leaves $\rightsquigarrow$ root, following the instruction structure
2. Decidability problems:
   - simplification and proof of formula
     undecidable in general, heuristics . . .
   - invariant generation
     undecidable in general, only specific invariant can be generated in some restricted conditions (i.e., inductive invariants)

# Accurracy vs Effectiveness trade-off

## Assertion language

| Theories | Complexity | Rappels |
|---|---|---|
| First order logic | undecidable | Interactive provers |
| Booleans | decidable | state enumeration |
| Intervals | quasi linear | approximation |
| Polyhedras | exponential | (better) approximation |

Tools:
Frama-C/WP (proofs), Frama-C/Value (intervals), Polyspace (polyhedras) . . .

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# A general framework : abstract interpretation

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. . . let’s jump to Dillig’s slides (from UT Austin, Texas) !

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# Analysis example: Value-Set Analysis

## Objective:

compute a (super)-set of possible values of each variable at each program location . . .

$Env(x, l)$ = value set of variable `x` at program location `l`

Several possible abstract domains to express these sets:

- bounded value sets (k-sets)
  ex: $Env(x, l) = \{0, 4, 9, 10\}, Env(y, l) = \{1\}, Env(z, l) = \top$
- intervals
  ex: $Env(x, l) = [2, 8], Env(y, l) = [-\infty, 7], Env(z, l) = [-\infty, +\infty]$
- differential bounded matrix (DBM)
  ex : $Env(l) = x - y < 10 \wedge z < 0$
- polyhedra (conjonction of linear equations)
  ex: $Env(l) = x + y \geq 10 \wedge z < 0$
- etc.

# VSA with intervals (example 1)

```
1. x := x+y ;
if x>0 then
     2. y:= x + 2
else
     3. y:= -x
4. fi
5. return x+y
```

Asumming (pre-condition) that:

$$x \in [-3, 3], y \in [-1, 5]$$

compute $Env(x, l)$ and $Env(y, l)$ for each program location $l$
what is the set of return values ?

# Computing intervals on expressions

## Syntax of expressions

$$e \to n \mid x \mid e + e \mid e \times e \mid \ldots$$

## Computation rules

$Val(e, Env)$ is the interval associated to $\mathtt{e}$ within $Env$

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$$Val(n, Env) \quad = \quad [n, n]$$

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$$\begin{aligned} Val(n, Env) &= [n, n] \\ Val(x, Env) &= Env(x) \end{aligned}$$

# Computing intervals on expressions

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$$e \to n \mid x \mid e + e \mid e \times e \mid \ldots$$

## Computation rules

$Val(e, Env)$ is the interval associated to `e` within $Env$

$$\begin{aligned}
Val(n, Env) &= [n, n] \\
Val(x, Env) &= Env(x) \\
Val(e1 + e2, Env) &= [a + c, b + d] \text{ where} \\
&\quad Val(e1, Env) = [a, b] \wedge Val(e2, Env) = [c, d]
\end{aligned}$$

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Val(n, Env) & = & [n, n] \\
Val(x, Env) & = & Env(x) \\
Val(e1 + e2, Env) & = & [a + c, b + d] \text{ where} \\
 & & \quad Val(e1, Env) = [a, b] \wedge Val(e2, Env) = [c, d] \\
Val(e1 \times e2, Env) & = & [x, y] \text{ where} \\
 & & \quad Val(e1, Env) = [a, b] \wedge Val(e2, Env) = [c, d] \\
 & & \quad\; x = min(a \times c, a \times d, b \times c, b \times d) \\
 & & \quad\; y = max(a \times c, a \times d, b \times c, b \times d)
\end{array}$$

# Intervals propagation

Propagation rules along the statement syntax:

- assignment

  $$\{Env1\}\ \mathrm{x} := \mathrm{e}\ \{Env2\}$$

  where

  $$Env2(x) = Val(e, Env1) \wedge Env2(y) = Env1(x) \text{ for } y \neq x$$

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- sequence

  $$\{Env1\}\ \texttt{s1}; \texttt{s2}\ \{Env2\}$$

  where

  $$\{Env1\}\ \texttt{s1}\ \{Env3\} \wedge \{Env3\}\ s2\ \{Env2\}$$

# Intervals propagation

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  where

  $$\{Env1\}\ \texttt{s1}\ \{Env3\} \wedge \{Env3\}\ s2\ \{Env2\}$$

- conditionnal

  $$\{Env\}\ \texttt{if}\ (\texttt{b})\ \texttt{then}\ \texttt{s1}\ \texttt{else}\ \texttt{s2}\ \{Env'\}$$

  where
  - {Env ∩ Val(b, Env)} `s1` {Env1}
  - {Env ∩ Val(¬ b, Env)} `s2` {Env2}
  - Env' = Env1 ⊔ Env2
    ($Env'(x)$ is the smallest interval containing $Env1(x)$ and $Env2(x)$, $\forall x$)

# Iteration ? (example 1)

```
1. x : = 0 ;
while (x < 2) do
  2. x := x+1
3. end
4. return x
```

compute *Env*$(x, l)$ for each program location $l$, where . . .

$$Env(x, 2) = Env(x, 1) \sqcup Env(x, 3)$$

# Iteration ? (example 1)

```
1. x : = 0 ;
while (x < 2) do
  2. x := x+1
3. end
4. return x
```

compute $Env(x, l)$ for each program location $l$, where . . .

$$Env(x, 2) = Env(x, 1) \sqcup Env(x, 3)$$

Actually, what we aim to compute is the least solution of function $Env$, i.e:

$$Env^0(\bot, l) \ \sqcup \ Env^1(\bot, l) \ \sqcup \ Env^2(\bot, l) \ \sqcup \ \ldots \sqcup Env^k(\bot, l) \ \sqcup \ \ldots$$

# Iteration ? (example 2)

```
1. x : = 0 ;
while (x < 1000) do
  2. x := x+1
3. end
4. return x
```

Compute $Env(x, l)$ for each program location $l$ . . .

# Iteration ? (example 2)

```
1. x : = 0 ;
while (x < 1000) do
  2. x := x+1
3. end
4. return x
```

Compute $Env(x, l)$ for each program location $l$ . . .

What happens if we replace `x := x+1` by `x := x-1` ?

# Iteration ? (example 2)

```
1. x : = 0 ;
while (x < 1000) do
  2. x := x+1
3. end
4. return x
```

Compute $Env(x, l)$ for each program location $l$ . . .

What happens if we replace `x := x+1` by `x := x-1` ?

How to cope with such **loooong**, or even **infinite**, computations ?

# Widening

For a lattice $(E, \leq)$, we define $\nabla : E \times E \to E$

$\nabla$ is a (pair-)widening operator if and only if

1. Extrapolation:

$$\forall x, y \in E.\ x \leq x \nabla y \wedge y \leq x \nabla y$$

# Widening

For a lattice $(E, \leq)$, we define $\nabla : E \times E \to E$

$\nabla$ is a (pair-)widening operator if and only if

1. Extrapolation:

$$\forall x, y \in E.\ x \leq x \nabla y \wedge y \leq x \nabla y$$

2. Enforce the convergence of $(Env(x, l))^{n \geq 0}$ by computing at each $l$ the limit of:

$$X_0 = \bot$$

$$X_i = \begin{cases} X_{i-1}, & \text{if } (X_{i-1}, l) \subseteq X_{i-1} \\ X_{i-1} \nabla Env(X_{i-1}, l), & \text{otherwise} \end{cases}$$

$(X_n)_{n \geq 0}$ is ultimately stationnary ...

→ $\boxed{\text{open “unstable” bounds (jumping over the fix-point) !}}$

# Widening on intervals

### Definition

$[a, b] \nabla [c, d] = [e, f]$ where,

- e = if $c < a$ then $-\infty$ else $a$
- f = if $b < d$ then $+\infty$ else $b$

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$[a, b] \nabla [c, d] = [e, f]$ where,

- e = if $c < a$ then $-\infty$ else $a$
- f = if $b < d$ then $+\infty$ else $b$

### Examples

- $[2, 3] \nabla [1, 4]$ ?
- $[1, 4] \nabla [2, 3]$ ?
- $[1, 3] \nabla [2, 4]$ ?

# Back to the previous example

```
1. x : = 0 ;
while (x < 1000) do
  2. x := x+1
3. end
4. return x
```

$$Env(x,2)_{n+1} = Env(x,2)_n \nabla (Env(x,1)_n \sqcup Env(x,3)_n)$$

$$\begin{aligned}
Env(x,2)_1 &= [0,0] \\
Env(x,2)_2 &= [0,1] \\
Env(x,2)_3 &= [0,999] \\
Env(x,3)_3 &= [0,1000]
\end{aligned}$$

→ stable solution . . .

# Back to the previous example

```
1. x : = 0 ;
while (x < 1000) do
  2. x := x+1
3. end
4. return x
```

$$Env(x,2)_{n+1} = Env(x,2)_n \nabla (Env(x,1)_n \sqcup Env(x,3)_n)$$

$$\begin{array}{rcl} Env(x,2)_1 & = & [0,0] \\ Env(x,2)_2 & = & [0,1] \\ Env(x,2)_3 & = & [0,999] \\ Env(x,3)_3 & = & [0,1000] \end{array}$$

→ stable solution . . . but not precise enough ?

$$Env(x,4)_3 = [1000,+\infty]$$

# Narrowing

lattice $(E, \leq)$, $\triangle : E \times E \to E$

$\triangle$ is a (pair-)narrowing operator if and only if

1. (abstract) intersection

$$\forall x, y \in E.\ x \cap y \leq x \triangle y$$

2. Enforce the convergence of $(Y_n)_{n \geq 0}$:

$$Y_n = \begin{cases} \lim X_i, & \text{if } i = 0 \\ Y_{i-1} \triangle Env(Y_{i-1}, I), & \text{otherwise} \end{cases}$$

   $(Y_n)_{n \geq 0}$ is ultimately stationnary ...

$\to$ $\boxed{\text{refines open bounds !}}$

# Narrowing on intervals

$[a, b] \bigtriangleup [c, d] = [e, f]$ where,

- e = if $a = -\infty$ then $c$ else $a$
- f = if $b = +\infty$ then $d$ else $b$

## Examples

- $[2, 3] \bigtriangleup [1, +\infty]$ ?
- $[1, 4] \bigtriangleup [-\infty, 3]$ ?
- $[1, 3] \bigtriangleup [+\infty, -\infty]$ ?

# Back (again !) to the previous example

```
1. x : = 0 ;
while (x < 1000) do
  2. x := x+1
3. end
4. return x
```

$$Env(x,2)_{n+1} = Env(x,2)_n \bigtriangleup (Env(x,1)_n \sqcup Env(x,3)_n)$$

$$\begin{array}{rcl} Env(x,3)_1 & = & [0,1000] \\ Env(x,4)_1 & = & [1000,+\infty] \\ Env(x,4)_2 & = & [1000,1000] \end{array}$$

→ stable solution . . .

# Outline

- Overview
- Principles
- Weakest Preconditions
- Abstract Interpretation
- Value-Set Analysis (VSA)
- Conclusion

# Challenges for static analysis

Accuracy vs scalability trade-off . . .

- inter-procedural analysis (+ recursivity . . . )
- multi-threading
- dynamic memory allocation
- modular reasonning
- libraries (+ legacy code)
- etc.

# Application to vulnerability detection ?

Clearly may provide some useful features:

- out-of-bounds array access
- arithmetic overflows
- incorrect memory access (null pointer, mis-aligned address)
- use-after-free
- etc.

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## But still some limitations:

- exploitability analysis (beyond standard program semantics) ?
- relevant and accurate memory model (for heap and stack)
- self-modifying code (e.g., malwares)
- binary code analysis (see next slide !)

# Static analysis on binary code

## Static analysis relies on a (clear) program semantics

- can be done at the assembly-level (or IR)
- but disassembling is undecidable . . .
- . . . and disassemblers may rely on static analysis !
  (to retrieve the program CFG)

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## Static analysis on low-level code is difficult

- no types (a single type for value, addresses, data, code, . . . )
- address computation is pervasive . . .
  ex: `mov eax, [ecx + 42]`
- function bounds cannot always be retrieved
  → many un-initialized memory locations
- scalability issues, e.g., complex but realistic memory model (≠ **independent** stack frames!)
- etc.

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“security analysis” = vulnerability detection

A pragmatic approach:

1. annotate the code with “vulnerability checks” (e.g., `frama-c -rte`) i.e., assertions to detect integer overflows, invalid memory accesses (arrays, pointers), etc

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**Rk:** some static analysis tools also provide bug finding facilities
(i.e., no false postives, . . . but false negatives instead)

# To summarize: some static analysis building blocks for security

## General purpose (but useful for security!)

- value analysis . . .
- data-flow analysis
  - statements **defining** a variable at a control location?
  - part of the code **impacted** by a given statement?
  - memory locations **assigned** by a given statement?
  - etc.

  ⇒ application on **program slicing**

  DEMO: frama-c impact analysis
- proof techniques (WP, theorem proving)

## More specificaly security-oriented

- non-interference
- constant-time programming
- pattern-based security checkers
- etc.

# Tool examples

## Disclaimer: non limitative nor objective list !

### Source-level tools

- Astrèe
- Coverity, **Polyspace**, CodeSonar, HP Fortify, VeraCode
- **Frama-C**, Fluctuat
- etc, etc, . . .

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- etc, etc, . . .

## Some binary-level tools

- x86-CodeSurfer
- VeraCode
- Angr
- **BinSec plateform**
- etc ?

**You can see also:**

- the NIST list of source code security analysers
- the Wikipedia List of static analysis tools