Licence Sciences et Technologies
Univ. Grenoble Alpes

Examen à mi-parcours du 7/11/22
INF 302 : Langages et Automates
L2, 2022/2023

Reminder about the instructions and some guidelines and remarks

- Duration : 2 hours.
- No exit before 30 minutes.
- No entry after 30 minutes.
- 3 sheets A4 recto and verso authorized.
- Any electronic device is prohibited (calculator, phone, tablet, connected watch, etc.).
- The quality of the copy will be taken into account. One point will be reserved for the quality of your copy.
- The grade is given as an indication.

* *

Write your name on each copy returned.
Indicate your group on each duplicate copy returned.

* *

Exercice 1 True or False.

Answer by true or false to the following questions. Justify carefully your answers (max 3 lines).

1. Given a language, one can find a deterministic finite-state automaton that recognises this language.
2. Given a regular language, one can find a deterministic finite-state automaton (without ϵ-transitions) that recognises this language.
3. If L is a finite-state language, then any language , $L' \subseteq L$ is a finite-state language.
4. If L is a finite-sate language language and L' is a finite language (i.e., a language with a finite number of words), then $L \cup L'$ must be a finite-sate language.
5. if L^* is finite-sate language then L is a finite-state language.

Exercice 2 Give a deterministic automaton.

1. Draw a finite deterministic automaton on the alphabet 0, 1 that recognises binary representations of integers divisible by 4.
2. Draw a deterministic finite automaton on the alphabet 0, 1 that recognises binary representations of integers divisible by 4 when read from the least significant bit.

Exercice 3 Determinisation, minimisation, regular expression.

1. Build a deterministic automaton that recognises $L(e)$ denoted by the regular expression $e = (bba)^*(\epsilon + b + bb)$.
2. Give the minimal deterministic automaton that recognises the complementary language of $L(e)$.
3. Give a regular expression defining the complementary of $L(e)$.
4. Let the language $L_{bb} = \{m \in \{a, b\}^* \mid bbm \in L(e)\}$. Show that L_{bb} is recognised by a deterministic automaton that you will give.

Exercice 4 ϵ-transitions, minimisation, intersection

Let $\Sigma = \{0; 1\}$ be the alphabet, we consider the two following languages :

— L the language of all the words over Σ^* containing aba ;

— M the language described by the regular expression $(b + aa^*bb)^*(\epsilon + aa^* + aa^*b)$

1. Give a non-deterministic automaton recognising L.
2. Give the minimal automata A recognising L.
3. Give a non-deterministic automaton with ϵ-transitions recognising M.
4. Give the minimal automata B recognising M.
5. Give the automata $A \times B$ recognising $L \cap M$.
6. Give the language recognised by $L(A \times B)$. What relation can be deduced between L and M.

Exercice 5

Let $e = \overline{(a^3)^*.(a^4)^*}$ be a regular expression over the alphabet $\{a\}$. The regular expression $\overline{e}$ describe the language $L(\overline{e}) = \Sigma^* \setminus L(e)$.

1. Give an automaton A such that $L(A) = L(e)$.
2. Is the the language $L(A) = \emptyset$, $L(A) \neq \emptyset$ and finite or infinite ?
3. If the language $L(A)$ finite give the set of the words.
4. Give an algorithm that takes as input a regular expression e and gives as output a regular expression $\overline{e}$ that describes the complementary language of denoted by e $(L(\overline{e}) = \Sigma^* \setminus L(e).)$.

Exercice 6 Finite-state language.

Let L, M be two languages and $LM^{-1} = \{v \mid \exists u \in M \ s.\ t.\ vu \ \in L\}$.

1. Prove that if L is a finite-state language then LM^{-1} whether M is a finite state or not