Licence Sciences et Technologies
Univ. Grenoble Alpes

Mid-term Exam
INF 302: Languages and Automata
L2, 2023/2024

Reminder about guidelines and some advices/remarks

— Duration : 2 hours (2 :00pm → 4 :00pm).
— No exit before 30 minutes.
— No entry after 30 minutes.
— 3 sheets of double-sided A4 paper are authorized.
— Any electronic device is forbidden (calculator, phone, tablet, etc.).
— **Care of your submission will be taken into account (-1 point if there is a lack of care).**
— Exercises are independent.
— The grading scale is indicative.
— The exam has 22 points, you need 20 points to get the maximal grade.

Exercice 1 (True or False - 5 points)

Answer by True or False to the following statements. Justify rigorously and concisely your answers (without proof). If you believe a statement is false, provide a counter example.

1. Any finite language is a finite-state language.
2. Any finite-state language is a finite language.
3. A deterministic and complete automaton recognizes the universal language.
4. If an automaton has a cycle, then its recognized language is infinite.
5. If an automaton has a reachable cycle, then the language it recognizes is infinite.
6. The difference between two finite-state languages is a finite-state language.
7. The concatenation of two finite-state languages is a finite-state language.
8. For an automaton A, we note $|A|$ the number of states in A. Let A_1 and A_2 be two DFAs. We have : $|A_1 \times A_2| \leq |A_1| \times |A_2|$.
9. If a language is a finite-state language, then any subset is a finite-state language.
10. If a language is a finite-state language, then any super set is a finite-state language.

Exercice 2 (Determinization, minimization - 4 points)

Let us consider $\Sigma = \{a, b, c\}$. Let us consider the automaton in Figure 1a.

1. Determinize the automaton.
2. Minimize the obtained automaton.

Exercice 3 (Determinization, minimization - 4 points)

Let us consider $\Sigma = \{a, b, c\}$. Let us consider the automata in Figures 1b and 1c.

1. Compute the product of these two automata.

Exercice 4 (A finite-state language ? - 3 points)

Let us consider $\Sigma = \{a, b\}$. Consider language L formed by words over Σ that contain as many occurrences of factor $a \cdot b$ as occurrences of factor $b \cdot a$.

1. Give two words in L and two words in $\Sigma^* \setminus L$.
2. Is L a finite-state language ? If yes, give an automaton that recognizes it, otherwise, explain informally why this language is not a finite-state language.

Exercice 5 Word formed by states taken in breadth-first traversal - 3 points

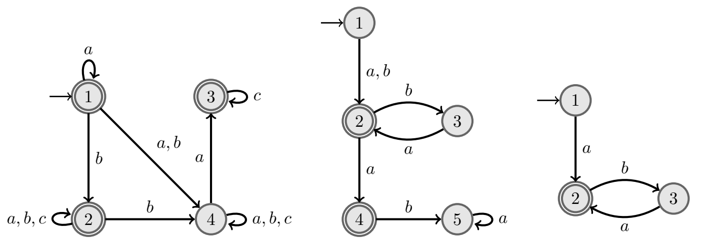
Let $(Q, \Sigma, q_0, \delta, F)$ be a deterministic finite-state automaton where all states are reachable. Let us suppose an order $\prec$ over symbols in Σ. We consider the words formed over the alphabet of states. These words are in the set Q^*. For example, if $Q = \{1, 2, 3\}$, then a word over Q is 32221.

1. Give an algorithm which for any automaton returns the word in Q^* of length $|Q|$ in which the order of symbols is the one in the order obtained by the breadth-first traversal where order $\prec$ is used to determine the priority between symbols.

Exercice 6 Depth of an automaton - 3 points

A path of an automaton any sequence of transitions starting from the initial state and following the transition function or relation (that is any two adjacent transitions in the sequence are such that the arrival state of the first is the starting state of the second). The length of a path is the number of elements in the sequence. The depth of a state is the length of the smallest path in the automaton that has this state as arriving state on the last transition. We call depth of an automaton the maximal depth of its states.

1. Give an algorithm which gives the depth of a state $q \in Q$ of a non-deterministic automaton $(Q, \Sigma, q_0, \Delta, F)$.
2. Give an algorithm which gives the depth of a non-deterministic automaton $(Q, \Sigma, q_0, \Delta, F)$.



(a) Automaton for Exercise 2 (b) Automaton for exercise 3 (c) Automaton for exercise 3

Figure 1 – Automata for the exercises